Zeno Dynamics in Quantum Statistical Mechanics

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14th May 2003 (v2)
by

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Experiments:

- (1990 Itano *et al.*) Transitions in $^9\mathrm{Be}^+$.
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Reviews:

- H Nakazato, M Namiki, S Pascazio (1996) Internat. J. Modern Phys. B **10** 247.
- M A B Whitaker (2000) Progress in Quantum Electronics **24** 1-106.

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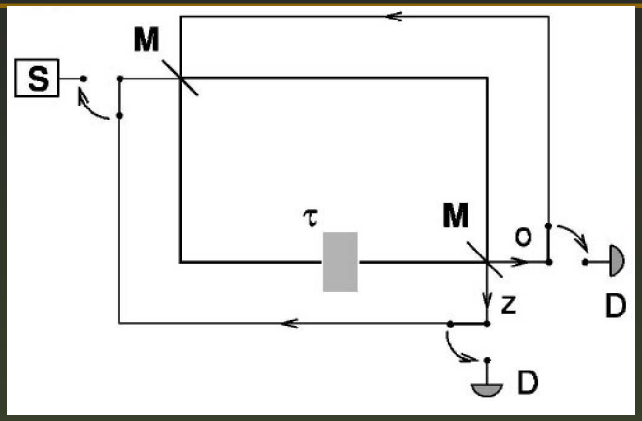

A Mach–Zehnder interferometer can be used to detect the presence of a black sample ($\tau = 0$), without absorbing the probing particle, if it is prepared to be in the Zeno channel (Z), and detected after L rounds in the interferometer, in the limit $L \to \infty$, see P Facchi *et al.* (2002) Phys. Rev. A **66** 012110 (pictures courtesy of S. Pascazio).

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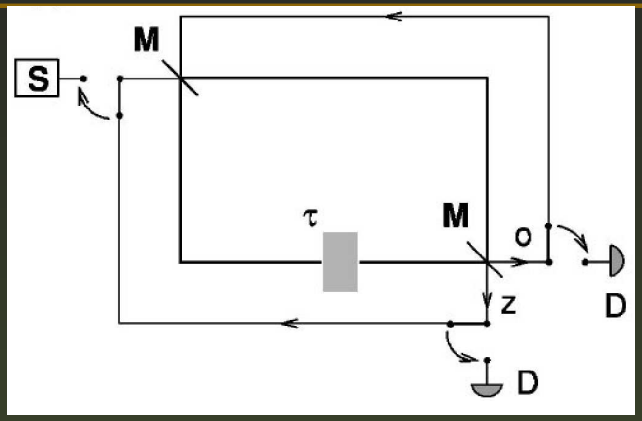

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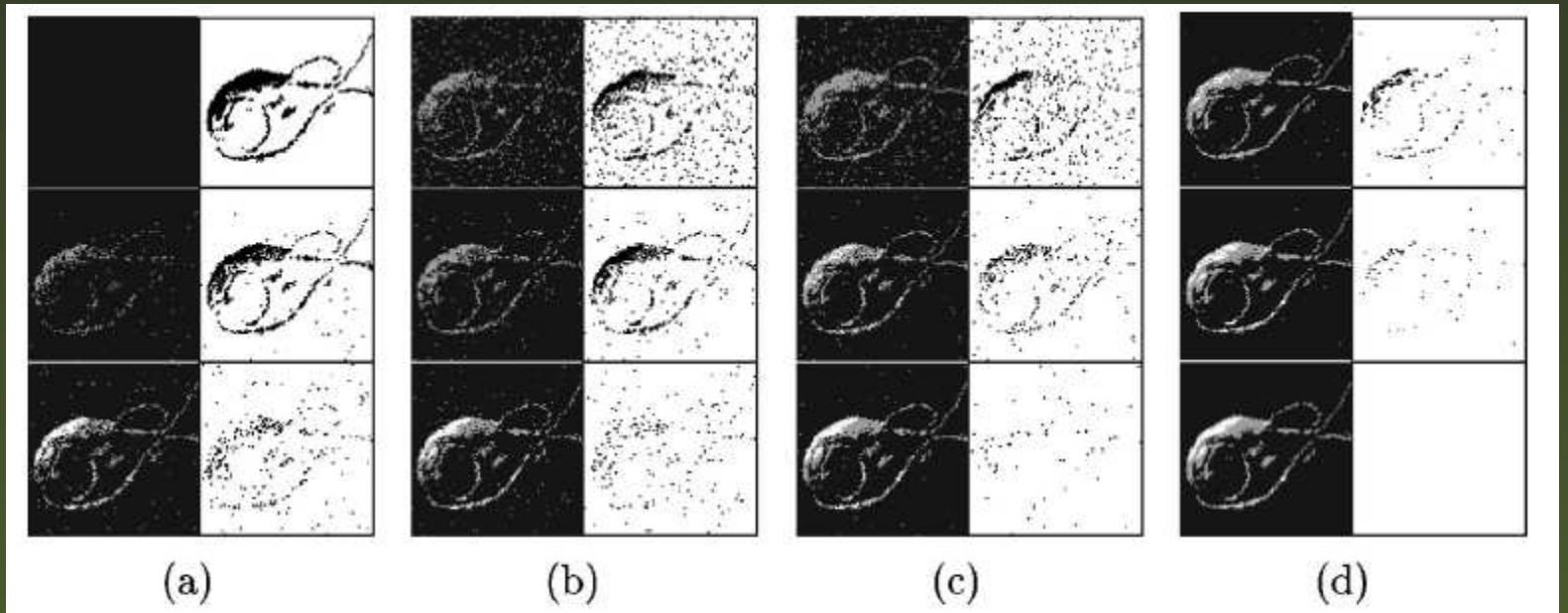

FIG. 5. Comparison of standard and Zeno tomographic techniques. In each frame, top left = reconstruction by standard technique; top right = misinterpreted pixels by the standard technique; center left = reconstruction by Zeno technique with $L=10$; center right = misinterpreted pixels by the Zeno technique with $L=10$; bottom left = reconstruction by Zeno technique with $L=165$; bottom right = misinterpreted pixels by the Zeno technique with $L=165$. The mean number of absorbed particles per pixel (irradiation) is $N_a=1.7$, 2.3, 4, and 13 for frames (a), (b), (c), and (d), respectively. The total number of particles N (total energy) scales approximately as $3N_a$, $1.8N_a$, and $6.5N_a$ for top, center, and bottom reconstructions, respectively. We used $\tau_w=0.99$, $\tau_g=0.96$, and $\tau_b=0.8$. The sample consists of 10 000 ($=100\times100$) pixels, where white, gray, and black occur with frequencies $\alpha_w=0.02$, $\alpha_g=0.07$, and $\alpha_b=0.91$, respectively. The number of misinterpreted pixels are (top to bottom) (a) 968, 786, 315; (b) 942, 596, 212; (c) 717, 382, 68; (d) 205, 69, 0.

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- Allows to identify the Zeno subspace to which the evolution will be (approximately) confined.
- The Zeno dynamics is identified as ordinary quantum dynamics with boundary conditions [Facchi, Pascazio, Scardicchio, Schulman (2001) Phys. Rev. A **65**].

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$$\omega_\beta(A) \stackrel{\text{def}}{=} \frac{\operatorname{Tr}_{\mathcal{H}}\left(\mathrm{e}^{-\beta H} A\right)}{\operatorname{Tr}_{\mathcal{H}}\left(\mathrm{e}^{-\beta H}\right)}, \quad \text{for } A \in \mathcal{A}, \ \dim \mathcal{H} < \infty.$$

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The proof follows [B Misra, E C G Sudarshan (1977) J. Math. Phys. **18**] using complex analysis. Differences: Holomorpy in a strip not in the upper halfplane, and $U(z)$ are unbounded (but with nice common core $\mathcal{A}\Omega$).

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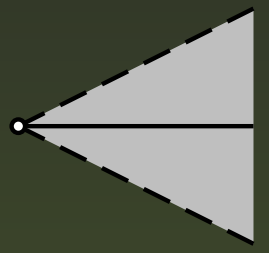

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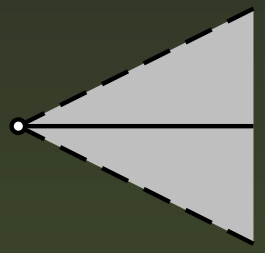

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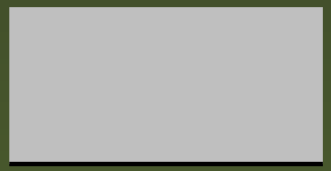

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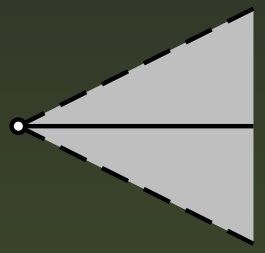

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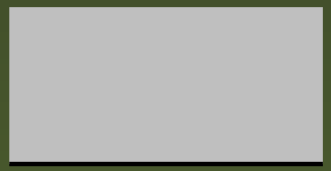

- **Quantum Statistical Mechanics (AUS):** The Zeno limit is needed on the boundary of a strip in $\mathbb{C}$. No sectorial result can be used.

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$$\left\| E^\perp U(\zeta) E A \Omega \right\| = C \cdot \|A\| \cdot \zeta$$

is valid for ζ *with* $|\zeta| < r_0$ *for some fixed* $r_0 > 0$ *and* $\operatorname{Im} \zeta \geq 0$*. Then*

i) *the strong operator limits* $W(t) \stackrel{\text{def}}{=} \text{s-lim}_{n\to\infty} \left[EU(t/n)E \right]^n$ *exist,*

ii) *form a strongly continuous group of unitary operators on the Zeno subspace* $\mathcal{H}_E \stackrel{\text{def}}{=} \overline{\mathcal{A}_E \Omega} \stackrel{\text{def}}{=} \overline{E\mathcal{A}E\Omega} \subset E\mathcal{H}$*,*

iii) *and the group* $W(t)$ *induces an automorphism group* τ^E *of* $\mathcal{A}_E$*.*

iv) *The vectors* $W(z)A_E\Omega$*,* $A_E \in \mathcal{A}_E$*, extend analytically to the strip* $0 < \operatorname{Im} z < \beta/2$ *and are continuous on its boundary.*

An Existence Theorem for Zeno Dynamics

Theorem. *Let* $(\mathcal{A}, \tau, \omega, \beta > 0, \pi_\omega, \mathcal{H}, \Omega, U, E)$ *as above. Assume that* (U, E) *satisfies* (AZC) *for* $\mathcal{A}$*:*

$$\left\| E^\perp U(\zeta) E A \Omega \right\| = C \cdot \|A\| \cdot \zeta$$

is valid for ζ *with* $|\zeta| < r_0$ *for some fixed* $r_0 > 0$ *and* $\operatorname{Im} \zeta \geq 0$*. Then*

i) the strong operator limits $W(t) \stackrel{\text{def}}{=} \text{s-lim}_{n\to\infty} \left[EU(t/n)E \right]^n$ *exist,*

ii) form a strongly continuous group of unitary operators on the Zeno subspace $\mathcal{H}_E \stackrel{\text{def}}{=} \overline{\mathcal{A}_E \Omega} \stackrel{\text{def}}{=} \overline{E\mathcal{A}E\Omega} \subset E\mathcal{H}$*,*

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- The (AZC) renders applicable the methods of perturbation theory!

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$\Longrightarrow$ This effect generalizes to projections onto τ^P-invariant subspaces ($\tau^P(E) = E$).

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From perturbation theory follows the uniform estimate

$$U_{\Lambda'}(t) = U_{\Lambda' \setminus \Lambda}(t) U_\Lambda(t) + \int_0^t U_{\Lambda' \setminus \Lambda}(\tau) U_\Lambda(\tau) W_\Phi(\Lambda; \Lambda') \mathrm{d}\tau + O(t^2). \qquad (\dagger)$$

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Although only applicable to semibounded Hamiltonians, this result yields a much sharper condition for the existence of Zeno dynamics than AZC, and a more general characterization of the Zeno dynamics: We had to pose (besides AZC) the stronger condition that $\mathcal{A}_E\Omega$ contains a dense set of analytic elements for *the original Hamiltonian* H.

Gibbs Equilibria for Zeno Dynamics

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Corollary. *If (U, E) satisfies (AZC) for $\mathcal{A}$, then, for every $\beta > 0$, the set of (τ^E, β)-KMS states of $\mathcal{A}_E$ coincides with the set of $(\widehat{\tau}^E, \beta)$-KMS states, where $\widehat{\tau}^E$ is induced by U_E.*

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Corollary. *Let $\beta > 0$. Let $\Lambda_\alpha \to \infty$ be such that the local dynamics converges uniformly, and the net of local Gibbs states ω_{Λ_α} has a thermodynamic limit point. If E_{Λ_α} converges in norm to a projection E in $\mathcal{A}$ satisfying (AZC), then $\omega_E(A_E) \stackrel{\text{def}}{=} \lim_\alpha \omega^G_{E_{\Lambda_\alpha}}(A_E)$ is a (τ^E, β)-KMS state on $\mathcal{A}_E$.*

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Let the interaction be such that the conditions of the Corollary are satisfied.
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This state is also the strong coupling ($\lambda \to \infty$) limit of the Gibbs equilibria for

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[M Fannes, R F Werner (1995) Helv. Phys. Acta **68** 635]

Example: The X-Y Model 1/3

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Global observable algebra $\mathcal{A} \stackrel{\text{def}}{=} \overline{\bigcup_{n \in \mathbb{N}} \mathcal{A}_{[-n,n]}}$.

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