Multi-level markets and incentives for information goods

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Abstract

The free-rider phenomenon which impedes the marketing of information goods is conventionally countered by copyright protection regulations and technology. Alternative ways to market information goods, in particular through systems based on the super-distribution of a good from buyer to buyer, have recently raised some interest. Some of them mimic peer-to-peer file-sharing networks, while advanced ones are mechanisms falling into the category of multi-level markets. Motivated by this, the present paper develops a general model for the monetary flux in a multi-level market, quantitatively describing the incentives that buyers receive through resale revenues. Based on it, some qualitative questions pertaining to a profitable marketing of information goods are discussed.

Key words: Information good, Multi-level market, Virtual good, Incentive, Copyright protection
JEL Classification: C51, C67, D4.

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1 Introduction

Information goods share the attributes of transferability and non-rivalry with public goods, and additionally are durable, i.e., show no wear out by usage or time. Like with a private good however, original creation can be very costly. The marketing of information goods is plagued by the problem of free riders, e.g., illegal copying and file-sharing on the Internet or over peer-to-peer networks. Conventional ways to counter "piracy" are copyright protection and digital rights management (DRM) measures. This practise raises controversy due

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to economical, policy, and pragmatic reasons. The literature on the economical effects of illegal copying and file-sharing is numerous, see, e.g., (Varian, 2000) and the references in (Gayer and Shy, 2003a). Oberholzer and Strumpf (2004) present an econometric study indicating that the effect of illegal file-sharing on record sales is negligible. Other studies support copyright protection, for instance Kinokuni (2003) shows theoretically that publisher's profits are maximised by a policy of restricting the number of copies to an optimal level if the distribution by (illegal) copies is more efficient than distribution by originals. Domon and Yamazaki (2004) examine the effects of illegal file-sharing on transaction and search costs from the perspective of optimal monopoly pricing. Lea and Hall (2004) survey the intellectual property controversy and the conflict with public interest. Yoon (2002) tries to determine an optimal level of copy-protection, considering social welfare losses due to underutilisation and underproduction. Negative implications of copy-protection and DRM from a consumer perspective are considered by Mulligan et al. (2003); Fetscherin (2003), while Schmidt et al. (2004) indicate technological challenges to DRM. Byers et al. (2004) elicit the multifarious vulnerabilities of copy-protection in the movie production and distribution process. The alternative of hardware taxation is investigated in (Gayer and Shy, 2003b).

In recent years, the content producing and marketing industries have undergone changes with the adoption of the Internet as a distribution channel, as already suggested in Shapiro and Varian (1999). The term *virtual goods*, coined by Aichroth and Hasselbach (2003), is used in information science for information goods in intangible, digital form, which are distributed through electronic networks, and we adopt it from now on. A current trend is to mimic the peer-to-peer (P2P) distribution channels which are commonly used by free riders for the marketing of virtual goods. In these approaches based on super distribution of goods from consumer to consumer, marginal costs for content distribution become negligible. Gayer and Shy (2003a) analyse how publishers can utilise P2P networks for the distribution of virtual goods. Antoniadis et al. (2004) point to economic inefficacies of P2P networks and suggest that incentives for participation may acquire importance.

The combination of P2P distribution with ideas derived from network marketing has recently led to the design of new market mechanisms as actual alternatives to copy protection and DRM. Their purport is a successful marketing of virtual goods even in the presence of freely available versions. They are based on incentives buyers receive upon reselling the good, while concurrently yielding a remuneration to its publisher.

Networked and multi-level marketing schemes are diversified. To be specific, the present paper focuses on a market with the following characteristics. The *originator* of the good is the monopolist holding the copyright for it and determining the sale and resale price for its legal distribution. Upon acquiring the good, a buyer also receives the right to resell it. Buyers and resellers are subsumed under the term *agents*. For each resale, part of the resale price is paid as a commission to the reseller, which is the economical incentive discriminating the legal version of the good from the free one, while the rest goes to a *collector* who may be a public agency collecting artist fees, a third business party, or identical with the originator. We call such a scheme applied to virtual goods and facilitated by electronic networks a *multi-level incentive management (MLIM) system*. The first MLIM system actually deployed and providing the main motivation for the present study is described by Grimm and Nützel (2002, 2003).

The most comprehensive economical systematisation of multi-level markets known to us is the second volume of the study (Micklitz et al., 1999) commissioned by the European Commission. Its first volume covers approaches to regulating policies for those markets. Theoretical literature on network marketing is scarce. Coughlan and Grayson (1998) present a model for the growth of retail network marketing organisations, focusing on detailed consideration of and matching to empirical parameters. Bhattacharya and Mehta (2000) argue on the basis of a socio-economic model that the success of network marketing organisations and the forming of closed social groups within them, often attributed to cult-like practises, may have a rational background. In view of this, the present paper contributes a building block to the presently lacking theoretical study of MLIM with focus on the marketing of virtual goods

Section 2 introduces a simple model for the monetary flux in a general multi-level market and derives the most basic results pertaining to it. Although conceived with virtual goods in mind, the model is applicable to a wider range of multi-level market schemes including those for physical goods. The most important qualitative traits of the markets described by the model are discussed in Section 3. This regards in particular the similarity to illicit schemes, the free-rider problem, network externalities, and the possibility to determine the incentive schedule by dynamical forward pricing. Section 4 presents conclusions and outlooks. The Appendix contains proofs of propositions.

2 Monetary Flux in a Multi-Level Market

The model we devise is continuous and kinematic. Firstly, all pertinent quantities are described by variables with continuous range. Secondly, the model describes the monetary flux between the market players, and other relevant quantities, such as the expected resale revenue, are to be derived from the kinematics.

No assumption is made on the decision making process of the agents and on the structure of the originator. Thus, the agents are solely discriminated by the time t at which they enter the market, i.e., buy the good from another agent already present in it. Consequently, buying the good happens only once for each agent, while resale can happen to arbitrary amounts at subsequent times. The market in turn is assumed to be homogeneous, i.e., all agents have equal probability to trade with each other. In accordance, no special market dynamics is assumed and the number $n(t)$ of agents in the market at time t is an unspecified function with continuous, non-negative, finite or infinite range. The resale price at time t is denoted by $\pi(t)$.

The expected (average) monetary incentive v_i for an agent entering the market at time t is given by

$$v_i(t) = v_r(t) - \pi(t), \tag{1}$$

that is, the expected revenue v_r from resale to agents entering the market at later times, diminished by the price at which the good was bought, i.e., the resale price at time t. To calculate v_r, note that the influx of agents into the market is given by $\dot{n}(t') = \mathrm{d}n(t')/\mathrm{d}t'$ at any later time $t' > t$, and if the agent was alone then one could integrate $\pi(t')\dot{n}(t')$ over an interval to obtain the resale revenue accumulated in it. But since there is competition in the reseller market, and all $n(t')$ agents have equal probability to strike a deal with the newcomers, the integrand must be divided by $n(t')$. Thus

$$v_r(t) = \int_t^\infty \frac{\gamma(t')\pi(t') - \tau}{n(t')} \dot{n}(t')\,\mathrm{d}t', \tag{2}$$

where the commission factor $0 \leq \gamma \leq 1$ accounts for the share of the resale price that an agent has to yield to the collector. The constant $\tau \geq 0$ subsumes the transaction cost an agent incurs in processing a single resale. Transaction costs incurred by buyers are absorbed in the price π for simplicity.

Reparametrisation by the monotonously increasing number of agents $n(t)$, makes the independence of the market dynamics manifest and yields

$$v_r(n) = \int_n^{n_\infty} \frac{\gamma(n')\pi(n') - \tau}{n'}\,\mathrm{d}n', \tag{3}$$

in which the market size n_∞ may be finite or infinite.

The model neither specifies all the endogenous and exogenous factors that may contribute to a multi-level market, nor does it presume any special estimators for them. Accordingly, the fundamental price function π, as well as the market dynamics, is left completely unspecified and can be generated by any underlying mechanism without affecting the general results derived from the model. Note that our understanding of the attribute “multi” in MLIM refers merely to the tree structure of the buyer graph, in contrast to the case that multiple subsequent buyer levels (or even all of them) contribute to the revenues of a buyer/reseller. The latter case is often realised in network marketing organisations for physical goods. The mentioned Potato system of Grimm and Nützel (2002, 2003) allows upstream payments from up to three following buyer levels. We argue that this is not desirable from a policy viewpoint in Section 3.1.

It is instructive to solve the homogeneous equation $v_i = 0$, $\gamma = 1$, $\tau = 0$, corresponding to an expected balance between resale revenues and buying price. In this case, π would necessarily have to satisfy the differential equation $\mathrm{d}\pi(n)/\mathrm{d}n = -\pi(n)/n$, the unique solution of which is $\pi(n) = \pi(0)/n$. With this solution however, one obtains $v_i = -\pi(0)/n_\infty$, showing that this π is not a solution of the homogeneous equation for $n_\infty < \infty$. The same reasoning applies to any constant, nonzero v_i and it follows that such a situation is not realisable in a finite market, due to the singular nature of the integral operator defining v_i. Thus it makes sense to specialise to finite markets, i.e., to take $n_\infty < \infty$. Then, a nonsingular re-parametrisation can be applied, replacing n with the market saturation $s = n/n_\infty$, $0 \le s \le 1$. The integral operator $K\colon \pi \longmapsto v_i$, a Volterra operator of the second kind, is defined by

$$(K\pi)(s) \overset{\text{def}}{=} v_i(s) = \int_s^1 \frac{\gamma(\sigma)\pi(\sigma)}{\sigma}\,\mathrm{d}\sigma + \tau\ln(s) - \pi(s). \tag{4}$$

As this operator describes a monetary flux between market agents, as well as from agents to collectors and into transaction costs, one would expect it to satisfy a balancing condition.

Proposition 1 *If π is bounded, then the incentive satisfies*

$$\int_0^1 v_i(\sigma)\mathrm{d}\sigma = \int_0^1 \big(\gamma(\sigma) - 1\big)\pi(\sigma)\mathrm{d}\sigma - \tau. \tag{5}$$

For bounded π, $\gamma = 1$, and $\tau = 0$ this law takes the form of a game-theoretical *zero-sum condition.*

$$\int_0^1 v_i(s)\,\mathrm{d}s = 0. \tag{6}$$

The zero-sum condition expresses that wins and losses in incentive compensate each other exactly. It is also the main reason why the attempt to obtain a nontrivial solution to the homogeneous equation was bound to fail (notice that $\pi = \pi(0)/n$ is too singular at $n = s = 0$ to fall in the scope of Proposition 1).

For regular enough π, the inverse of K is easily obtained as a solution of the inhomogeneous equation $K\pi = v_i$. We denote the derivatives of π, v_i, with $\dot\pi$, $\dot v_i$, respectively.

Proposition 2 *Let $0 \le \gamma(s) \le 1$ be in $C^\infty([0,1])$ and $\tau < \infty$. Define*

$$\check{K}v_i(s) \overset{\text{def}}{=} -E_\gamma(s)^{-1}\int_0^s \dot v_i(\sigma)E_\gamma(\sigma)\,\mathrm{d}\sigma + \frac{\tau}{\gamma(s)}. \tag{7}$$

with the abbreviation $E_\gamma(s) \overset{\text{def}}{=} \exp(\int_0^s \gamma(\sigma)/\sigma \mathrm{d}\sigma)$*. Let* $\mathcal{W}$ *be the subspace of functions* $v_i \in C^1((0,1])$ *satisfying i)* $v_i(s) = o(1/s)$, $\dot{v}_i(s) = O(1/s)$ $(s \to 0)$*, ii)* $v_i(1) = -\check{K}v_i(1)$*, iii)* $\int_0^1 v_i(\sigma)\mathrm{d}\sigma = \int_0^1 (\gamma(\sigma) - 1)\check{K}v_i(\sigma)\mathrm{d}\sigma$*. Then* K *maps* $\mathcal{V} \overset{\text{def}}{=} C^1([0,1])$ *bijectively onto* $\mathcal{W}$ *with inverse* $\check{K} \colon \mathcal{W} \longrightarrow \mathcal{V}$*.*

Although nothing in principle prevents a monetary flow from earlier market entrants to later ones by negative prices, the conventional case is that of positive resale prices. We characterise it for $\gamma = 1$ and vanishing transaction cost τ. According to the inversion formula in Proposition 2, it is sufficient that $\dot{v}$ is non-positive for π to remain non-negative, that is positive (non-negative) prices are always obtained if the incentive is (strictly) monotonic decreasing. Yet, a sharp condition can be formulated as follows.

Proposition 3 *Let* $\pi \in C^1([0,1])$*,* $\gamma = 1$*,* $\tau = 0$*. Then,* π *is positive if and only if*

$$\frac{1}{s}\int_0^s v_i(\sigma)\,\mathrm{d}\sigma > v_i(s) \quad \textit{for all } s. \tag{8}$$

This result has a rather direct interpretation. It says that the monetary flow is always directed backwards if and only if the expected incentive at a certain time is smaller than the average expected incentive before that time.

Figure 1 a) shows the most basic example of resale revenues and incentives resulting from a constant price. It exhibits the logarithmic singularity present in the continuous model, and which will always emerge if $\pi(0)$ is positive. The singularity is avoided if $\pi(0) = 0$ as in b) and c). Additionally, in c) the incentive is forced to zero as $s \to 1$ by letting π approach zero, and also shows a case where v_i is not always monotonic decreasing and π is still positive. The effect of a commission factor is exhibited in Figure 1 d).

A continuous model is an idealisation of a realistic market where buyers enter one by one, i.e., the market size evolves in discrete steps. This entails artifacts, most notably the logarithmic singularity for $v_i(s)$ as $s \searrow 0$ when $\pi(0) > 0$. Therefore one needs to examine the discrepancy between the incentive obtained from the continuous model and the one calculated by discrete summation somewhat more closely. For a constant price $\pi(s) = \pi$, the discrete model can be solved directly. Agents are labeled with $k = 1, \dots, n_\infty$, by the order of market entrance, and this yields for the expected incentive $\overline{v}_i$ of the discrete case

$$\overline{v}_i = \pi \left(\sum_{k'=k+1}^{n_\infty} \frac{1}{k'-1} - 1 \right) = \pi \left(\Psi(n_\infty) - \Psi(k) - 1 \right), \tag{9}$$

where the Digamma function $\Psi(z) = \Gamma'(z)/\Gamma(z)$ is the logarithmic derivative of the Gamma function, see (Olver, 1997, p. 39).

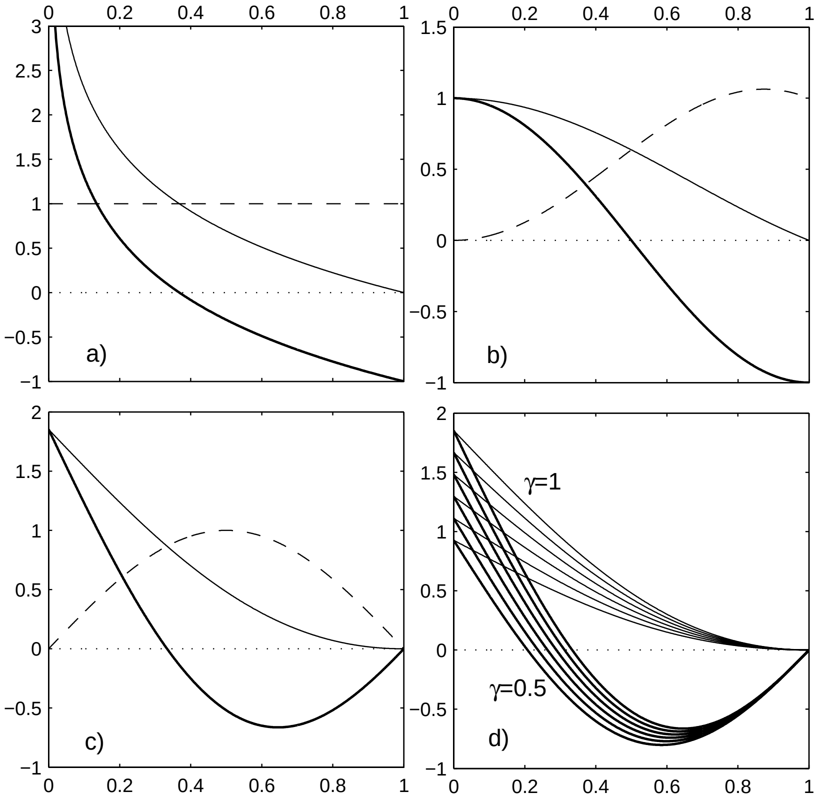

Fig. 1. Examples for prices π (dashed), expected resale revenues v_r (thin solid), and incentives v_i (thick solid). For a)—c) $\gamma = 1$. a) $\pi = 1$, $v_r = -\ln s$, $v_i = -\ln s - 1$. b) $\pi(s) = \sin(\pi s)/(\pi s) - \cos(\pi s)$, $v_r = \sin(\pi s)/(\pi s)$, $v_i = \cos(\pi s)$. c) $\pi(s) = \sin(\pi s)$, $v_r = \mathrm{Si}(\pi) - \mathrm{Si}(\pi s)$, $v_i = \mathrm{Si}(\pi) - \mathrm{Si}(\pi s) - \sin(\pi s)$, where Si is the integral sine function. d) Price as in c) with commission factor γ varying from 1 to 0.5 in steps of size 0.1.

In the general case, we have to look at the difference between $v_i(s)$ and the discrete incentive $\overline{v}_i(s \cdot n_\infty)$ at the corresponding point.

Proposition 4 *For bounded, non-negative π, $\gamma = 1$, $\tau = 0$, holds*

$$|v_i(s) - \overline{v}_i(sn_\infty)| \leq \frac{\pi_{\max}}{2}\left[\frac{1+s}{sn_\infty} + \frac{1}{6}\frac{1+s^2}{(sn_\infty)^2} + O\left(\frac{1+s^4}{(sn_\infty)^4}\right)\right], \tag{10}$$

with $\pi_{\max} \overset{\text{def}}{=} \max_{s\in[0,1]} \pi(s)$, and in which the term of order $(sn_\infty)^4$ is strictly dominated by the previous one.

Te error behaviour of the continuous model is rather benign in that it decays with the inverse of the market size at any finite saturation $s > 0$. For fixed $k = sn_\infty$ on the other hand, a constant error bounded by $c_k\pi_{\max}$ for some $c_k > 0$, will always remain.

3 Discussion

3.1 Similarity to Illicit Schemes

Multi-level marketing carries negative connotations and is illegal in special forms known as pyramid selling, snowball systems, chain-letters, etc., under many jurisdictions. (Micklitz et al., 1999, Vol. II) present criteria to distinguish between legitimate multi-level marketing and such practises that are to be considered illicit. In view of them, five arguments can be produced in favour of the legitimacy of multi-level marketing of virtual goods in general, and the MLIM systems within the scope of the present model in particular. First, illicit schemes often require resellers to keep a large, non-returnable stock of the good. The effect of this kind of inventory loading is however not present in the case of virtual goods, due to the very nature of information goods. Second, also the marginal costs for their replication and redistribution are mostly orders of magnitude smaller than resale prices and thus transaction costs are largely insignificant. Third, the compensation plans of illicit schemes often emphasise recruitment of personnel over resale of the good. This is not the case in MLIM where incentives are strictly bound to individual sales of a virtual good of positive pecuniary value. In other words, every agent can at least expect a rebate on the price paid for the good through resale revenues. Fourth, our present model does not allow for down-line payments. Only the directly succeeding level $n+1$ of agents to which the good is directly sold contributes to the revenues of agents at level n. Although, down-line payments for resale are not seen as problematic by (Micklitz et al., 1999, Vol.II, page 236), we would argue that they shift incentive payment too much from individual sales efforts to uncontrollable market dynamics. Finally, realistic information on achievable revenues is crucial for the legitimacy of multi-level marketing. The present MLIM model offers in principle the possibility to determine and publish the price and incentive schedule in advance, cf. Section 3.5.

3.2 The Free-Rider Problem

The effectiveness of MLIM in countering illegal copying of a virtual good can be theoretically analysed based on the present model, since it is agnostic with respect to underlying communication and distribution networks. Thus we can consider the duopoly situation of two goods distributed in the same MLIM system, one of them being the free version with $\pi = 0$, whereas the other one is marketed by the originator at a positive price. Assume that each agent decides rationally and only based on monetary values for one of them at market entrance. If the price schedule $\pi(s)$ for the legal version is *public knowledge* agents can calculate the schedule of expected incentives $v_i(s)$. Depending on agents' information about the total number of potential buyers and the current market sizes for the legal and the free-rider version, a nonzero part will estimate to obtain a negative v_i and decide to acquire the free version. This reduces the potential market size for the legal good by a certain, positive amount. If furthermore the price schedule is *common knowledge*, the remaining agents will adjust their estimates on resale revenues accordingly. Some of them will end up with negative estimated incentives and also opt for the free version. This inductively reduces the number of buyers of the legal version to zero. By similar reasoning, also common knowledge of merely the zero-sum condition (6) would preclude any success of the legal good over the free-riders.

The viability of MLIM in the presence of free versions is therefore a genuine question of information economy, hinging on the distinction between public and common knowledge about the market mechanism and the price schedule as the determining exogenous variable. The analysis above in fact also pertains to pure chain-letters which do not distribute a good of value at all. Their mere existence makes it clear that in multi-level markets some knowledge on mechanism and resulting externalities for buyers must be assumed to be only public or even private in a realistic model. In view of (Coughlan and Grayson, 1998; Bhattacharya and Mehta, 2000), resale revenues are perhaps not the only contribution to a good's individual utility in a general multi-level market. A dynamical model for competition of goods in MLIM markets must take other factors for agents' decisions into account. The next section alludes to specific network externalities which can contribute to them.

It is conceivable that the incentive through resale revenues is insufficient to make MLIM effective against free riders. Originators could combine it with copy protection, or otherwise discriminate the legal version in the MLIM system from illegal copies distributed over P2P networks, e.g., by added value.

3.3 Network Externalities

It is to be expected that markets based on super distribution, in particular P2P and MLIM systems for virtual goods, are related to network externalities. They can be endogenously produced in those markets as well as influence the market's dynamical growth. Network effects are understood in the literature as the benefit that accrues to a user of a good or a service because he or she is one of the many who use it. Simple functional forms of network effects for special types of networks, e.g., telecommunication networks, such as Sarnoff's, Metcalfe's, and Reed's law, are often taken as heuristics to explain the dynamics of the growth of networks of the respective type. The most prominent phenomena traced back in this way to network effects are a "slow startup", the existence of a “critical mass” (Lim et al., 2003), and strong (supra-exponential) growth after this mass has been reached. Models for network externalities and their effects on prices and utility are numerous and detailed, see, e.g., Economides (1996a,b) and references therein, while global models, such as Swann (2002) for the possible functional forms of network externalities, are scarce.

Network utility can spatially be understood as the *aggregate* value, summed over all members of the network, or as the *individual* value enjoyed by single members. In models depending on a dynamical parameter, each case is in turn subdivided on the temporal axis into the *dynamic* utility given as a function of the saturation s, as a relative variable, and the *kinematic* utility, which is the scaling behaviour of the utility with the market size n_∞.

The only kinematic aggregate utility arising is that obtained by the replication of the good and redistribution of it, a contribution which is always of order $O(n_\infty)$, like in broadcast networks. The incentive contributes to aggregate utilities only in a dynamic way, since it is given by

$$n_\infty \cdot \int_0^s v_i(s')\,\mathrm{d}s', \tag{11}$$

which approaches zero for $s \to 1$, respectively is of the order $O(-n_\infty)$, more precisely $-n_\infty \int_0^1 (\gamma(\sigma) - 1)\pi(\sigma)\mathrm{d}\sigma$, by (5), if a commission is in effect.

The only contribution to the dynamic, individual utility is v_i, since the kinematic, individual utility, i.e., the scaling behaviour of v_i with n_∞, is $O(1)$ precisely if π is $O(1)$ ($n_\infty \to \infty$), i.e., if the price stays bounded. This *scale-free* property, which becomes manifest in the continuous limit $n_\infty \to \infty$, is a salient feature of the model presented. It is not an artifact of the continuous idealisation, since the error bound (10) shows that it is stable for nonzero s. However, for small, fixed $k = sn_\infty$, and if $\pi(0) > 0$, a scaling of the kinematic, individual utility of order $O(\ln n_\infty)$ appears (meaning that in pyramid schemes profiteers gains scale logarithmically with the number of participants).

In conclusion, the incentive is the only network externality to the agents in the MLIM market, except for a logarithmic, kinematic effect on early buyers. This was to be expected since the market described has no special structural properties. However in a more detailed model describing a competitive, duopoly or oligopoly MLIM market for virtual goods, other structural effects must be taken into account. Domon and Yamazaki (2004) point out that increased search costs due to unauthorised file-sharing can bias markets for virtual goods and impede the distribution of legal copies. We call such those positive effects, which depend on the relative dominance of one good in a MLIM market over others, *multiplier effects*. It is conceivable that they are significant for the dynamics of competition in those markets.

3.4 Market Inhomogeneities

Multiplier effects are a prominent example for details which undermine one crucial assumption underlying the model presented, namely *homogeneity* of the market. Uniform agents in a structureless market are a good approximation if the number of potential participants is large and consists of a rather homogeneous group of individuals, for instance with special personal preferences, e.g., musical. However, if the market is biased in the sense that there is a group of agents with systematically higher trading capacities, the assumption breaks down. In reality, large music labels running direct sale web sites are a counterexample where it is violated. On the other hand, inhomogeneities and multiplier effects carry the imminent danger that an MLIM market can be cannibalised at an early stage by an agent with overwhelmingly high communication capacity, e.g., a popular web site, who could then obtain a practical monopoly. The study of Maurer and Huberman (2003) indicates that monopoly creation could be a rather natural effect in E-commerce. While the originator of the good is not too affected by this phenomenon if a commission model is used, other buyers' incentives are always negatively affected. To what extent the market can be levelled by means of the IM system, e.g., by providing equal communication capacities to all participants, restricting or controlling resale volumes or frequencies, etc., warrants separate discussion.

3.5 Dynamical Forward Pricing

A new option arising from the model presented is the possibility, via the inversion formula (7), to dynamically adapt the incentive during the evolution of the market if the originator controls the price as an external parameter. Such **MLIM systems with dynamical forward pricing** can be used to design actual market mechanisms. Dynamical forward pricing is not a new concept for information goods, (cf. Jagannathan and Almeroth, 2004), but has not been widely considered in the context of multi-level markets, neither for virtual nor physical goods.

Figure 1 shows basic possibilities for price functions. The constant price in a) is associated with a strong favouritism of early buyers, and increasingly penalises later ones. A typical example for what is conventionally termed an early subscriber discount is shown in b). Such a price schedule is often used as an initial invitation to enter, i.e., a means to spur the distribution of the good in an early stage, for instance to counteract a slow startup effect. This can become important to counter free-riders, since in their presence early buyers cannot be sure about their potential resale revenues which depend logarithmically on the market size (remember that $v_i(k = sn_\infty)$ scales as $\ln n_\infty$). The price associated with the incentive in b) is monotonously increasing, thus later buyers pay more and receive less incentiv, and are thus disfavoured. Example c) improves on b) by letting the price vanish when the market reaches saturation. This v_i combines an early subscriber discount with a rebate for late adopters who finally obtain the good gratuitously, a price schedule which can spur the distribution of the good in late phases, when it may have lost individual utility, e.g., due to dwindling popularity. Assuming that the market has a positive, endogenous growth dynamics in an intermediate phase associated with a high demand, it is reasonable to let the prices peak and lower the incentive in this phase, as in c). Deepness and position of the minimum of v_i can be adjusted almost arbitrarily. Finally, d) shows the effect of a commission on the incentive. In particular it can be seen that the point at which the incentive becomes negative is not significantly shifted with decreasing γ.

For an implementation of dynamical forward pricing information becomes essential, in particular the current size $n(t)$ of the market must be known. This is the case when a central server counts every single acquisition of the good, as, e.g., realised in the mentioned Potato system. The market size n_∞, necessary to calculate the saturation $s = n/n_\infty$, is more difficult to determine. Though it could be estimated by market research, comparison with earlier runs of the system for different goods, or other means of educated guessing, a more pragmatic solution suggests itself. As in Figure 1 c) and d), setting the price to zero after some finite time, respectively at an *a priori* given n_∞ obtains a natural condition for *closure* of the market. Though running counter to the aim of maximising the diffusion of the good, the effect on the achievable turnover is limited if the price becomes small enough at high saturations.

Mixed forms of dynamical price settings can be envisaged, e.g., correlation of π with the buying frequency, combined with a frequency or price threshold below which the price is set to zero and the market closed. In any case, designing the optimal price schedule is a complex task, in particular in competitive situations. Then arises, for instance, the additional difficulty that the total market saturation for all goods cannot be determined by a single party.

4 Conclusions

We showed how a basic model for the monetary flux in an idealised multi-level market can be applied to examine market mechanisms for the distribution of virtual goods. In particular, the role of incentives through resale revenues as alternatives to copyright protection becomes amenable to an analytical discussion. Section 3 showed how some intriguing features pertaining to MLIM markets can be examined based on, and in terms of the model.

However, in view of its idealised and simplified nature, the model should be considered as a starting point for further model building and theoretical analysis. The latter includes a proper placement of MLIM in the framework of the principal-agent model (Laffont and Martimort, 2002), in particular for a rigorous discussion of the free-rider problem in terms of moral hazard, optimality, and equilibria. A prerequisite is a detailed micro-model for agents' decision processes in a duopoly MLIM market, taking into account goods' differences in individual utility (popularity), bounded rationality (cf. Section 3.2), and network externalities such as multiplier effects. To complement theoretical approaches by simulation studies of the dynamics of MLIM markets, using the methods of agent-based computational economics (Tesfatsion, 2001), such as Zerfiridis and Karatza (2004), can be helpful. They would allow to study the influence of particularities (topologies) of the communication network underlying the market, and address the issue of inhomogeneities. In conclusion, the present model may give rise to some fecund directions for further research.

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Appendix: Proofs of Propositions

PROOF. (of Proposition 1) We let $\tau = 0$ since its contribution to the incentive in (4) just integrates to $-\tau$ as stated. For $\varepsilon > 0$ calculate $\int_\varepsilon^1 v_r$ as

$$\begin{aligned}\int_\varepsilon^1 \mathrm{d}\sigma \int_\sigma^1 \frac{\gamma(\sigma')\pi(\sigma')}{\sigma'}\,\mathrm{d}\sigma' &= \int_\varepsilon^1 \mathrm{d}\sigma' \int_\varepsilon^{\sigma'} \frac{\gamma(\sigma')\pi(\sigma')}{\sigma'}\,\mathrm{d}\sigma \\ &= \int_\varepsilon^1 \frac{\sigma'-\varepsilon}{\sigma'}\gamma(\sigma')\pi(\sigma')\,\mathrm{d}\sigma' \\ &= \int_\varepsilon^1 \gamma(\sigma')\pi(\sigma')\,\mathrm{d}\sigma' - \varepsilon \int_\varepsilon^1 \frac{\gamma(\sigma')\pi(\sigma')}{\sigma'}\,\mathrm{d}\sigma'.\end{aligned}$$

If $\pi(s)$ is bounded on $[0, 1]$ as assumed then the second term is of order $O(\varepsilon \ln \varepsilon)$ and vanishes as $\varepsilon \searrow 0$. The first term converges to $\int_0^1 \gamma(s)\pi(s)\,\mathrm{d}s$.

PROOF. (of Proposition 2) It suffices to consider the case $\tau = 0$. For $\pi \in \mathcal{V}$, $(K\pi)(s)$ is a continuously differentiable function in the interval $(0, 1]$ with derivative $-\gamma(s)\pi(s)/s - \dot{\pi}(s)$. The latter is of order $1/s$ as $s \searrow 0$ since π stays bounded at zero and γ is bounded. For the same reason, the integral in $K\pi$ is $O(\ln s)$ $(s \to 0)$, which is $o(1/s)$, showing $(K\pi)(s) = o(1/s)$ $(s \to 0)$ and thus condition i). Due to these two asymptotic conditions we can apply $\check{K}$ to obtain a continuously differentiable function on $[0,1]$:

$$
\begin{aligned}
(\check{K}K\pi)(s) &= \check{K}\left(\int_\sigma^1 \frac{\gamma(\sigma')\pi(\sigma')}{\sigma'}\,\mathrm{d}\sigma' - \pi(\sigma)\right) \\
&= -E_\gamma^{-1}(s)\int_0^s \left(-\frac{\gamma(\sigma)\pi(\sigma)}{\sigma} - \dot{\pi}(\sigma)\right) E_\gamma(\sigma)\,\mathrm{d}\sigma \\
&= E_\gamma^{-1}(s)\left(\int_0^s \frac{\gamma\pi(\sigma)}{\sigma} E_\gamma(\sigma)\,\mathrm{d}\sigma - \int_0^s \pi\dot{E}_\gamma(\sigma)\,\mathrm{d}\sigma + \Big[\pi E_\gamma(\sigma)\Big]_0^s\right).
\end{aligned}
$$

This becomes, using $\dot{E}_\gamma(\sigma) = \gamma(\sigma)E_\gamma(\sigma)/\sigma$

$$
= E_\gamma^{-1}(s)\left(\pi(s)E_\gamma(s) - \lim_{\sigma\to 0}\pi(\sigma)E_\gamma(\sigma)\right) = \pi(s),
$$

since, for smooth γ and bounded π, $\lim_{\sigma\to 0}\pi(\sigma)E_\gamma(\sigma) = 0$. Now, since $\check{K}v_i = \pi$ for $v_i = K\pi$, Proposition 1 shows that $v_i = K\pi$ satisfies condition iii) and also ii), since clearly $v_i(1) = -\pi(1)$. Therefore $K\pi \in \mathcal{W}$. On the other hand, if $v_i \in \mathcal{W}$ then $\dot{v}_i = O(1/\sigma)$ $(\sigma \to 0)$ by i), and the last calculation showed that $\check{K}$ can be applied to it and obtains a differentiable function in $(0,1)$ which extends continuously to $[0,1]$. That is $\check{K}v_i \in \mathcal{V}$ and we calculate

$$
(K\check{K}v_i)(s) = -\int\limits_s^1 \frac{\gamma(\sigma')}{E_\gamma(\sigma')\sigma'}\int\limits_0^{\sigma'} E_\gamma(\sigma)\dot{v}_i(\sigma)\mathrm{d}\sigma\mathrm{d}\sigma' + \frac{1}{E_\gamma(s)}\int\limits_0^s E_\gamma(\sigma)\dot{v}_i(\sigma)\mathrm{d}\sigma.
$$

Applying partial integration to the first term with $\int \gamma(\sigma')/(E_\gamma(\sigma')\sigma') = -1/E_\gamma$ yields

$$
\begin{aligned}
&= -\int\limits_s^1 \dot{v}_i(\sigma')\mathrm{d}\sigma' + \left[\frac{1}{E_\gamma(\sigma')}\int\limits_0^{\sigma'} E_\gamma\dot{v}_i(\sigma)\mathrm{d}\sigma\right]_s^1 + \frac{1}{E_\gamma(s)}\int\limits_0^s E_\gamma\dot{v}_i(\sigma)\mathrm{d}\sigma \\
&= v_i(s) - v_i(1) + \frac{1}{E_\gamma(1)}\int\limits_0^1 E_\gamma(\sigma)\dot{v}_i(\sigma)\mathrm{d}\sigma.
\end{aligned}
$$

The integral is nothing but $-\check{K}v_i(1)$ and from property ii) we see that this equals $v_i(1)$. It follows $(K\check{K}v_i)(s) = v_i(s)$ as desired.

PROOF. (of Proposition 3) Partial integration yields

$$
\int_0^s \sigma\dot{v}_i(\sigma)\,\mathrm{d}\sigma = \int_0^s v_i(\sigma) - sv_i(s),
$$

where we have used that $\sigma v_i(\sigma) \to 0$ for $\sigma \searrow 0$ if π is C^1, by property i) in Proposition 2. The result follows upon inserting the above equation into the inequality $\pi(s) > 0$ and using Proposition 2.

PROOF. (of Proposition 4) We have for sn_∞ integer

$$\begin{aligned}|v_i(s) - \overline{v}_i(sn_\infty)| &= \left| \int_s^1 \frac{\pi(s')}{s'}\,\mathrm{d}s' - \sum_{k'=sn_\infty+1}^{n_\infty} \frac{\pi(k'/n_\infty)}{k'-1} \right| \\ &= \lim_{\varepsilon \searrow 0} \left| \int_s^{1+\varepsilon} \pi(s') \left(\frac{1}{s'} - \sum_{k'=sn_\infty+1}^{n_\infty} \frac{\delta(s'-k'/n_\infty)}{s'} \right) \mathrm{d}s' \right|\end{aligned}$$

where we extended π continuously in a small interval $[1, 1+\varepsilon]$, and used Dirac's δ-function to incorporate the sum in the integral. Now, the non-negative factor π can be drawn out to estimate

$$\begin{aligned}&\le \pi_{\max} |\Psi(sn_\infty) - \ln s - \Psi(n_\infty)| \\ &= \pi_{\max} \Big(|\Psi(sn_\infty) - \ln sn_\infty| + |\ln n_\infty - \Psi(n_\infty)| \Big)\end{aligned}$$

Using the asymptotic expansion of the Ψ function for r a positive integer, see (Olver, 1997, p. 295), we obtain

$$|\ln r - \Psi(r)| \le \frac{1}{2r} + \sum_{m=1}^{n} \frac{|B_{2m}|}{2mr^{2m}}, \quad \text{for } n \ge 0,$$

where B_{2m} is the $2m$-th Bernoulli number. From this follows the claim.

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